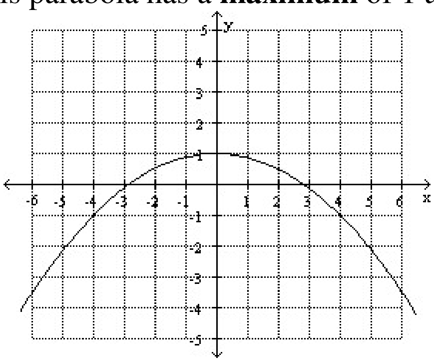
Math 112 – Section 2.4: Graphs of Quadratics

$f(x) = ax^2 + bx + c$ (Parabola)

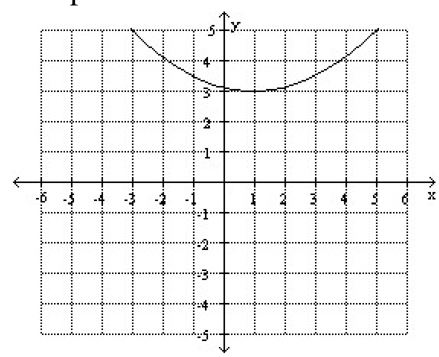
**Standard form:** $f(x) = a(x-h)^2 + k$

where the **vertex** is $(h, k)$, which is the highest or lowest point,
**axis of symmetry** is the line $x = h$
**opens up** if a > 0 (has a minimum point)
**opens down** if a < 0 (has a maximum point)

This parabola has a **maximum** of 1 at x = 0.



This parabola has a **minimum** of 3 at x = 1.



There are 3 ways to find the vertex:

1) Finding the vertex from the standard form
2) Using the formula $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right) = \left(\dfrac{-b}{2a}, \dfrac{4ac-b^2}{4a}\right)$ if not in standard form.
3) Completing the square if not in standard form ← a time consuming method

**Example 1:** Find the vertex of each.

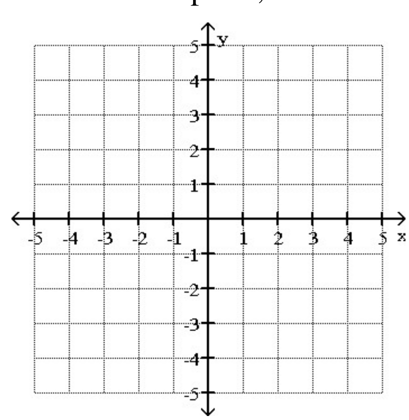
a) $f(x) = x^2 + 10x + 23$

b) $f(x) = (x-2)^2 + 3$

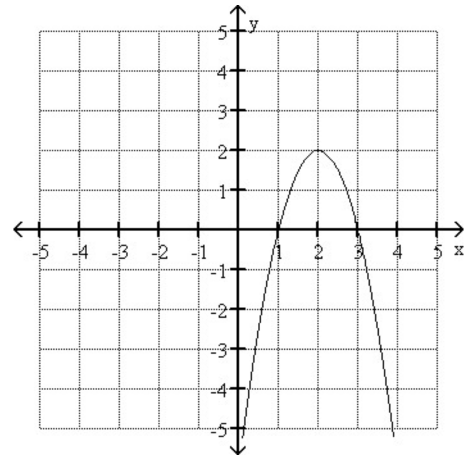
**Example 2**: Find the vertex, axis of symmetry, the minimum or maximum point, intervals where increasing and decreasing, and graph each.

a) $f(x) = (x+3)^2 - 1$

Vertex = (– 3, – 1)
Axis of symmetry: x = – 3
Minimum point (– 3, – 1)
Increasing: $(-3, \infty)$
Decreasing: $(-\infty, -3)$



b) $f(x) = -2x^2 + 8x - 6$




**Applications:**

**Example 3: Profit**

Total Profit = Total Revenue – Total Cost

P(x) = R(x) – C(x)

Find the maximum profit and the number of units that must be sold in order to yield the maximum profit for $R(x) = 20x - 0.1x^2$ and $C(x) = 4x + 2$.

**Solution:**

$P(x) = R(x) - C(x) = (20x - 0.1\,x^2) - (4x + 2) = -0.1\,x^2 + 16x - 2.$

$$\text{The vertex} = \left(\frac{-16}{2(-0.1)}, P\left(\frac{-16}{2(-0.1)}\right)\right) = (80, P(80))$$

$$= (80, -.01 \cdot 80^2 + 16 \cdot 80 - 2) = (80,\ 638)$$

Thus, the number of units that must be sold is 80, and the maximum profit is \$638.

**Example 4: Height of a stone**

A stone is thrown directly upward from a height of 30 ft with an initial velocity of 60 ft/sec. The height of the stone t seconds after it has been thrown is given by the function $s(t) = -16t^2 + 60t + 30$. Determine the time at which the stone reaches its maximum height and find the maximum height.