MATH 151, FALL 2011
COMMON EXAM II - VERSION A

Last Name: ______________________________ **First Name:** ______________________________

Signature: ______________________________ **Section No:** ______________

PART I: Multiple Choice (15 questions, 4 points each. No Calculators)
Write your name, section number, and version letter (A) of the exam on the ScanTron form.

1. Find the derivative of $h(t) = (t^4 + 7t)^5$.

   (a) $h'(t) = 5(4t^3 + 7)^4$
   (b) $h'(t) = 5(t^4 + 7t)^4(4t^3)$
   (c) $h'(t) = 5(t^4 + 7t)^4(4t^3 + 7)$
   (d) $h'(t) = 20t^{19} + 7^5(5t^4)$
   (e) $h'(t) = 5(t^4 + 7t)(4t^3 + 7)$

2. If $f(x) = \sin(g(x))$, find $f'(2)$ given that $g(2) = \frac{\pi}{3}$ and $g'(2) = \frac{\pi}{4}$.

   (a) $\frac{\sqrt{3}\pi}{8}$
   (b) $\frac{\pi}{8}$
   (c) $\frac{1}{2}$
   (d) $-\frac{\sqrt{3}\pi}{8}$
   (e) $-\frac{\pi}{8}$

3. At what point on the graph of $f(x) = \sqrt{x}$ is the tangent line parallel to the line $2x - 3y = 4$?

   (a) $\left(\frac{9}{16}, \frac{3}{4}\right)$
   (b) $\left(\frac{9}{16}, 0\right)$
   (c) $\left(\frac{4}{3}, \frac{2}{\sqrt{3}}\right)$
   (d) $\left(\frac{16}{9}, \frac{4}{3}\right)$
   (e) $\left(\frac{1}{16}, \frac{1}{4}\right)$

4. A ball is thrown vertically upward with a velocity of 80 feet per second and the height, s, of the ball at time t seconds is given by $s(t) = 80t - 16t^2$. What is the velocity of the ball when it is 96 feet above the ground on its way up?

   (a) 112 ft/sec
   (b) 48 ft/sec
   (c) 16 ft/sec
   (d) 24 ft/sec
   (e) 64 ft/sec

5. Given the equation $2xy + \pi \sin(y) = 2\pi$, find $\frac{dy}{dx}$ when $x = 1$ and $y = \frac{\pi}{2}$.

(a) $-\frac{\pi}{2-\pi}$

(b) $-\frac{\pi}{3}$

(c) $-\frac{\pi}{2+\pi}$

(d) 0

(e) $-\frac{\pi}{2}$

6. Find the equation of the tangent line to the graph of $f(x) = \frac{x}{1+2x}$ at $x = 1$.

(a) $y - \frac{1}{3} = -\frac{1}{9}(x-1)$

(b) $y - \frac{1}{3} = -\frac{4}{9}(x-1)$

(c) $y - \frac{1}{3} = \frac{x}{1+2x}(x-1)$

(d) $y - \frac{1}{3} = \frac{1}{9}(x-1)$

(e) $y - \frac{1}{3} = -\frac{x}{1+2x}(x-1)$

7. Which of the following vectors is tangent to the curve $\mathbf{r}(\mathbf{t}) = \left\langle \sqrt{t^2+1}, t \right\rangle$ at the point $(2, \sqrt{3})$?

(a) $\left\langle \frac{\sqrt{3}}{2}, 1 \right\rangle$

(b) $\left\langle \frac{1}{2}, 1 \right\rangle$

(c) $\left\langle \frac{\sqrt{3}}{4}, 1 \right\rangle$

(d) $\left\langle \frac{2}{\sqrt{5}}, 1 \right\rangle$

(e) $\left\langle \frac{1}{\sqrt{5}}, 1 \right\rangle$

8. Find the 81^{st} derivative of $f(x) = \dfrac{1}{x}$.

(a) $f^{(81)}(x) = -\dfrac{(81)!}{x^{81}}$

(b) $f^{(81)}(x) = \dfrac{(80)!}{x^{80}}$

(c) $f^{(81)}(x) = \dfrac{(81)!}{x^{81}}$

(d) $f^{(81)}(x) = -\dfrac{(80)!}{x^{80}}$

(e) $f^{(81)}(x) = -\dfrac{(81)!}{x^{82}}$

9. Find the linear approximation, $L(x)$, for $f(x) = \sqrt[3]{x}$ at $x = -8$.

(a) $L(x) = -2 + \dfrac{1}{12}(x+8)$

(b) $L(x) = -2 - \dfrac{1}{12}(x+8)$

(c) $L(x) = -2 + \dfrac{1}{12}(x-8)$

(d) $L(x) = -2 - \dfrac{1}{12}(x-8)$

(e) $L(x) = -2 + \dfrac{1}{4}(x+8)$

10. $\lim\limits_{x \to -\infty} (9 - 7e^{-x}) =$

(a) ∞

(b) 0

(c) $-\infty$

(d) 7

(e) 9

11. $\lim\limits_{x \to 0} \dfrac{\sin^3(4x)}{x^3} =$

(a) ∞

(b) 4

(c) 1

(d) 0

(e) 64

12. Given $f(x)$ is a one-to-one function, find $g'(3)$ where g is the inverse of the function $f(x) = x^9 + x^3 + x$.

(a) $g'(3) = \dfrac{1}{12}$
(b) $g'(3) = 1$
(c) $g'(3) = \dfrac{1}{9}$
(d) $g'(3) = \dfrac{1}{13}$
(e) $g'(3) = 13$

13. Find the slope of the tangent line to the curve $x = t^2 + t + 1$, $y = \sqrt{t} + 4$ at $t = 9$.

(a) 114
(b) $\dfrac{3}{5}$
(c) $\dfrac{19}{6}$
(d) $\dfrac{5}{12}$
(e) $\dfrac{1}{114}$

14. Find the derivative of $f(x) = x^3 e^{2x}$.

(a) $f'(x) = 6x^2 e^{2x}$
(b) $f'(x) = 3x^2 e^{2x} + 2x^3 e^{2x}$
(c) $f'(x) = 3x^2 e^{2x} + x^3 e^{2x}$
(d) $f'(x) = 3x^2 e^{2x}$
(e) $f'(x) = 3x^2 e^{2x} + 2x^4 e^{2x-1}$

15. Find the derivative of $g(x) = \dfrac{x^3 + 1}{x^2 + 1}$.

(a) $g'(x) = \dfrac{x^4 + 3x^2 - 2x}{x^2 + 1}$
(b) $g'(x) = \dfrac{5x^4 + 3x^2 + 2x}{x^2 + 1}$
(c) $g'(x) = \dfrac{x^4 + 3x^2 + 2x}{(x^2 + 1)^2}$
(d) $g'(x) = \dfrac{x^4 + 3x^2 - 2x}{(x^2 + 1)^2}$
(e) $g'(x) = \dfrac{5x^4 + 3x^2 + 2x}{(x^2 + 1)^2}$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

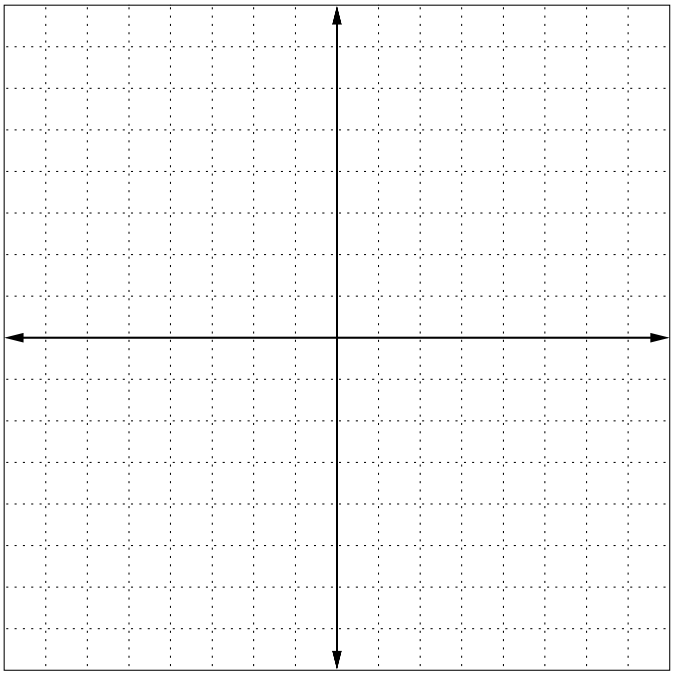
16. (8 pts) An observer is standing 10 feet from the base of a balloon launching point. At the instant the balloon has risen vertically 5 feet, the height of the balloon is increasing at a rate of 8 feet per minute. How fast is the distance from the observer to the balloon changing at this same instant? Assume the balloon starts on the ground and rises vertically.

17. (8 pts) Find the second derivative of $f(x) = \tan(x^2)$.

18. (8 pts) A rain gauge has the shape of a cone with the vertex at the bottom whose radius is half of the height. Given that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, find the differential dV in terms of only h and the differential dh. Use the differential dV to estimate the change in volume when the height of water in the gauge increases from 6 cm to 6.2 cm.

19. (8 pts) For the equation $y = e^{2x} + e^{-3x}$, show $y'' + y' - 6y$ is a constant. Find the constant.

20. (8 pts) Draw a diagram to show there are two tangent lines to the parabola $y = 2x^2$ that pass through the point $(1, -2)$ by sketching the parabola and both tangent lines on the grid provided below. Find the x-coordinates where these tangent lines touch the parabola.



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Question	Points Awarded	Points
1-15		60
16		8
17		8
18		8
19		8
20		8
		100