Modeling of Normalized Frequency Gradient for Fused Single Mode Fiber of Coupling Ratio

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Abstract- **Although the coupling ratio research has shown good progress in experimental and theoretical calculation, coupled waveguide fibers still have power reflection and power losses due to the effects of fabrication. Two fibers are coupled and heated by injecting hydrogen flow at a pressure of 1bar with the torch flame in the range of 800-1350C. During the fusion process some optical parameters are found to vary over a wide range. A coupling coefficient is estimated from experimental result of coupling ratio distribution ranging from 1% to 75%, and a refractive index is calculated from the empirical calculation. It is found that the change of the fiber geometry affects normalized frequency even for single mode fiber.**

Coupling ratio as the function of coupling coefficient and separation of fiber axis changes the normalized frequency at the coupling region. Normalized frequency is derived from the radius, wavelength and refractive index parameters. At the left and right side of the coupling region, some parameters are decreased and increased respectively. At the center of the coupling region, some optical parameters are assumed to remain constant.

The normalized frequency is integrated over the pulling length in the range of 7500-9500μm for 1-D where radial and angle directions are ignored. Simulation result shows that the normalized frequency is significantly affected by the radius compared to the wavelength and refractive index of the core and cladding. This simulation has a dependence of power phenomena in transmission and reflection for communication and industrial application of coupled fibers.

Keywords: Single mode fiber, coupling ratio, coupling coefficient, normalized frequency

I. INTRODUCTION

Recently, many applications of fiber have been used for the industrial instruments, public utilities, government, researchers, and even for home usage in terms of both passive and active devices. At the same time, development and fabrication of coupled fibers have been widely expanded for tunable filter and optical waveguide switch in obtaining effective and efficient applications. The coupling fibers can also be applied for multipurpose usage in the telecommunications and as device sensor [1,2]. To split, divide, combine and control information channel of power transmission, waveguide fibers are usually fabricated and coupled from one fiber to another to form as a junction. It is then used to obtain information system through wave and power distribution from one junction to another for certain purposes. These processes produce good results as shown by the output and input of the system. However, coupling fiber fabrications applied as a source and waveguide should also consider parametric function emerging along the processes when information transfer to fibers occurs. This would complicate the problem, particularly at the junction since the source; electric field and power are affected by the waveguide, structure and geometry of the fiber itself. Therefore development of these must be kept monitored regularly to obtain good resolutions in transmitting and receiving the output information through power after passing the junction.

Coupled fiber used with far field distance of communication has less effect of reflection at a junction. However, given the situation of some junctions and near field distance, the loss of transmission power gives significant effect in delivering power ratio. One of main phenomena occurring to the optical couplers, such as the coupling of mode in space [3,4] which contributes to the power propagation along the coupled fiber is termed as coupling coefficient. Coupling coefficient can be expressed as an effective power range that transmits to another fiber. The separation between the two fibers is considered significant in the coupling coefficient, as it determines the effective power transmission to another fiber. The value of coupling coefficient can be determined by experimental and theoretical calculations. Even though the determination of coupling coefficient for a practical directional coupler looks complicated, it can however be solved by evaluating the channel waveguide modes and observing the fiber geometry, where some calculation of coupling coefficient range can be obtained. The electric wave that propagates along the cylindrical fiber runs with a small interference with "slow" variation of the amplitude mode, which means that it satisfies condition [5] of $|d^2/dz^2 A_k| << |\beta_k\ d/dz\ A_k|$ with a parabolic approximation where β_k, z, and A_k are propagation constant, direction propagation and amplitude of the wave respectively. Power transmission depends on the distribution of the coupling ratio with a fractional power which mainly occurs at coupling region. The coupling region itself has three regions which are based on core and cladding geometry located at the left, center and right. At the center of the coupling region is the main coupling where the power propagation splits from one core to another at a near distance through cladding.,

In fusion process, heating coupled fibers are not homogenous in changing their structures and geometries at coupling region. Their changes are complicated due to the uncertainty in the refractive indices and fiber geometries as a result of the coupling ratio. However, they tend to decrease along fibers from one edge to the center of the coupling region and then, increase again to the other end. It also occurs that the wave and power propagate partially at the propagation direction even though the conservation of power is independent of such position. Single mode fiber is used to propagate this power. It guides only one mode and can be seen from normalized frequency. Normalized frequency is dimensionless depending on the core radius, wavelength, the core and cladding refractive index. To investigate the coupling region as a function of the coupling ratio, the normalized frequency is simply derived and modeled. This is to study the mode and its dependency to normalized frequency parameter. Therefore, this paper describes the normalized frequency gradient as being computed from the coupling coefficient range and coupling region data which is obtained from coupling ratio distribution experimentally.

II. COUPLED FIBER AFTER FUSION: COUPLING COEFFICIENT AND REFRACTIVE INDEX

The Single mode fiber (SMF-28e®) is coupled by two fibers with the same geometry which is assumed to be homogeneous, isotropic materials with a very small gradient of refractive index along propagation [6,7]. The fibers are heated and are slightly unstable flame torch at a temperature range of 800-1350C. The core after the fusion is reduced from 80.5% to 94%. A half pulling length of fiber coupler increases significantly over the coupling ratio. The coupling length overrides the coupling ratio due to the longer time take at the coupling region by a few ms to reach complete coupling power. The coupling ratio distribution is more affected by coupling coefficient and it cannot determine the cladding diameter to be constant even though the LP_{01} diameter size is achieved. It is of course, the decrease of the refractive index of the junction fibers to reach the coupling ratio, while the 2 cores distance is closer than the radius of those two claddings. Consider a coupled identical single mode fiber 1X2 splits one source into two transmission lines turning as a Y junction.

$$\left.\begin{aligned} P_a(z) &= P_o - P_b(z) \\ P_b(z) &= P_o\,\kappa^2/(\kappa^2+\delta^2)\,\sin^2[(\kappa^2+\delta^2)^{1/2} z] \end{aligned}\right\} \quad (1)$$

P_o is an input power as a laser diode source λ =1310nm to guide a complete power transfer in a distance of $z=m\pi/\kappa$; $m=0,1,2,\ldots,$ for $m=1$, $z=L_c=\pi/\kappa$. L_c is the coupling length in millimeter unit. The axial length is then periodically changed by a coupling ratio [6] whereas $P_b/(P_a+P_b)$ and $P_a/(P_a+P_b)$ are respectively defined as coupling and transmission power. The wave propagates as a sinus and cosine wave where $\kappa = \sqrt{(\delta^2+\kappa^2)}$ is the coupling coefficient and δ which is the phase mismatch factor defined as $(\beta_1-\beta_2)/2$. If $\delta=0$, it then has equal phase velocities in both modes and $\kappa/(\kappa^2+\delta^2)$ is a fraction of power exchanged. A simple empirical relationship is used to calculate the value κ [8] which is as follows:

$$\kappa = (\pi/2)\,(\sqrt{\delta}/a)\,\exp\,[-(A + B\,\bar{d} + C\,\bar{d}^2)] \quad (2)$$

where $A = 5.2789 - 3.663V + 0.3841V^2$ $\quad C = -0.0175 - 0.0064V - 0.0009V^2$

$B = -7769 + 1.2252V - 0.012V^2$ $\quad \delta = (n_1^2-n_2^2)/n_1^2$; $\bar{d} = d/a$

n_1 and n_2 are respectively the core and cladding refractive index.. It is experimentally measured in the range of 0.9-0.6/mm in correspondence to the determination of refractive index by the empirical equation to the core and cladding which is n_1=1.4640-1.4623 and n_2=1.4577-1.4556 respectively, for coupling ratio 1-75%. The separation of fibers between two cores, 2d cannot be precisely obtained, but for the calculation purpose, the mean value is set at the range of 10-10.86μm. It may then be concluded that the empirical calculation to reach the coupling ratio can only be detected and imposed by power.

IV. NORMALIZED FREQUENCY GRADIENT MODEL

SMF-28e® has a dominant mode, LP_{01} with V=2.405. When coupled fibers are being fused and pulled, the values changes according to the wavelength source and characteristic of the fibers. At the coupling region with z direction V as a function of $V(r, n, \lambda)$, can then be linearly defined as [8],

$$\nabla V = \nabla[(2\pi a/\lambda)\,(n_1^2 - n_2^2)^{1/2}] \quad (3)$$

where the changes of each parameter are due to the structural and geometrical properties of fibers. Fiber sizes are decreased from the left and right side of the coupling region. At the center of the coupling region, they are assumed linearly constant.

Consider the pulling length of fibers as follows, $P_L = PL_1 + PL_2 + PL_3$,, where $PL_1 = PL_3$ and $PL_2 = C_L$ (C_L is coupling length). Supposing fibers are imposed in z direction, derivation of equation (3) is then given by

$$\nabla V \equiv [\, \nabla(a) + \nabla(\lambda) + \nabla(n_1) + + \nabla(n_2)] \quad (4)$$

Assume the derivation of each parameter for $\nabla V|_+$ and $\nabla V|_-$ is defined as

$$\nabla(a)|_k \equiv \Delta a / 2\Delta(P_L - C_L) \;;\; \nabla(n_1)|_k \equiv \Delta n_1 / 2\Delta(P_L - C_L);\; \nabla(\lambda)|_k \equiv \Delta\lambda / 2\Delta(P_L - C_L) \quad \nabla(n_2)|_k \equiv \Delta n_2 / 2\Delta(P_L - C_L) \quad (5)$$

The subscript of k is (+), (0) or (-) whereas $\nabla V|_+$ expresses positive gradient when the radius of fibers is decreased and it turns to negative gradient when the radius of fibers is increased at $\nabla V|_-$. At PL_2, it is assumed that $\nabla V|_0 \approx 0$. For a simplified normal frequency model where the fibers are set at the certain temperature level and the change of fiber properties homogenous, it is then considered that at PL_1 the value of a linearly changes as same as n_1 and n_2 towards the temperature. Meanwhile the wavelength depends linearly upon n_1 and n_2. Those parameters change similarly at PL_3 but in opposite sign.

The first and second terms of equation (4) are evaluated with the boundary limit that is combined to equation (5) resulting in,

$$\left.\begin{aligned} \nabla V|_+ = \nabla V|_- = \nabla V|_{+,-} &= \{(2\pi/\lambda)(n_1^2 - n_2^2)^{1/2} da/dz + 2\pi a (n_1^2 - n_2^2)^{1/2} d(1/\lambda)/dz\, d\lambda/dz \\ &\quad + (2\pi a/\lambda)[\tfrac{1}{2}(n_1^2 - n_2^2)^{-1/2}(2n_1\, dn_1/dz - 2n_2\, dn_2/dz)\}\,|_{+,-} \\ \nabla V|_0 &= 0 \end{aligned}\right\} \quad (6)$$

Since the fibers are similarly fused and pulled to the left and the right side, then $\nabla(a)|_+ = \nabla(a)|_-$, $\nabla(\lambda)|_{+n1} = \nabla(\lambda)|_{-n1}$, $\nabla(\lambda)|_{+n2} = \nabla(\lambda)|_{-n2}$, $\nabla(n_1)|_+ = \nabla(n_1)|_-$, and $\nabla(n_2)| = \nabla(n_2)|_+$. Given the range of the coupling region in order to calculate V and to correct dV/dz in determining the effect of change of fibers geometrically then equation (6) can be derived and be kept constant. Assume total V can be written by $V = \sum_{k=(+,0,-)}^{3} V_k$ and $\frac{dV}{dz} = \sum_{k=(+,0,-)}^{3} \nabla V_k$

where $\nabla V_k = \nabla V|_+ + \nabla V|_0 + \nabla V|_- = 0$. When total ∇V is not constant hence $\frac{1}{\nabla V}\frac{dV}{dz} = \frac{1}{z}$

where z represents the position. By multiplying both sides with V_k and $\frac{1}{\nabla V}$ for normalization of ∇V, the equation then becomes

$$\frac{V_k}{\nabla V}\frac{dV}{dz} = V_k \left[\sum_{k=(+,0,-)}^{3} \frac{1}{\nabla(2\pi a (n_1^2 - n_2^2)^{1/2})} \right] \frac{d}{dz}(V_+ + V_o + V_-) \quad (7)$$

Therefore, to keep total of ∇V constant, finally we combine the two terms of equation (6) and (7) for V_k, obtaining

$$\left(\frac{dV_k}{dz}\right)_{correction} = \left(\frac{dV}{dz}\right) - \frac{V_k}{\nabla V}\frac{dV}{dz} \quad (8)$$

This formula describes that during power propagation at the coupling region the total ∇V remains constant even though V_k changes. For illustration purposes, this model can then be depicted in Figure 1.

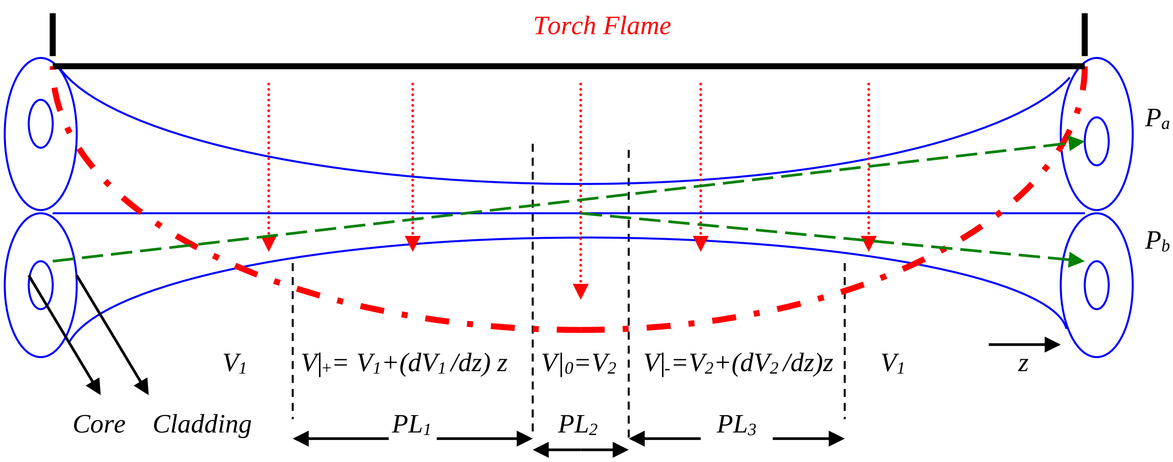

Figure 1. SMF-28e® coupler fiber is heated by H_2 gas at temperature 800-1350C. The initial core and cladding diameter are respectively 8.2µm and 125µm. They reduce 75-94% in size after fusion. Total pulling of fibers to the left and right side is in the range of 7500-9500µm with the velocity of ≈100µm/s. Pulling is stopped subject to the coupling ratio achieving a setting value.

V. INTEGRATION OF NORMALIZED FREQUENCY AND DISCUSSION

The values of V partially change at the coupling region are integrated over z direction of core radius and a half pulling length. It is then run by Ode45 Matlab program with a set of input data for the refractive index of core and cladding, wave length and initial V. For the given values of equation (5), the result at PL_1 shows as follows:

$$\left.\begin{aligned} &\nabla(a)|_{+} = 1044.3864 \text{ to } 796.8127 \times 10^{-6}, && \nabla(n_1)|_{+} = 1.05 \text{ to } 1.65 \times 10^{-6}, \\ &\nabla(\lambda)|_{+n1} = -0.0006542 \text{ to } -0.0010101 \times 10^{-9}, && \nabla(n_2)|_{+} = 1.35 \text{ to } 2.05 \times 10^{-6}, \\ &\nabla(\lambda)|_{+n2} = -0.0008376 \text{ to } -0.0012823 \times 10^{-9}, && \end{aligned}\right\} \quad (9)$$

Those parametric values emerge as the result of the coupling ratio that is set in the range of 1 to 75%. It has a function of coupling coefficient which then produces the parametric values gradients existing in that number range. The value of $\lambda_o/\lambda=n$ moves to decrease along PL_1 until it meets coupling length and then inversely increases along PL_3. The equation (9) is similar to PL_3 but the gradient in opposite sign.

The graph of ∇V at PL_1, as calculated from equation (6) in Figure 2, is the normalized frequency gradient at the first and the end of coupling region as depicted in Figure 1. Supposing that ∇V is constant then from equation (8), it also gives the same curve compared to equation (6). Comparing the two curves, it shows that the change of V does not occur as much as that shown at the end of PL_1. It shows that at the end of PL_1, it has decreased significantly due to the fact of pulling the length and heating of fibers at the end of PL_1. Again it also means that the higher gradient depicted to reach the coupling length results to greater reflection power to the source fiber and crosstalk fiber due to the refractive indices gradient and greater loss of power from the core to the cladding and to edge of the cladding due to the radius gradient. This result has the same values for PL_3 but in the negative gradient. The normalized frequency gradient does not make parametric change to reduce or add power significantly along coupling region neither at PL_1 nor at PL_3, since the total power remains constant but it is partially affected by the normalized frequency parameters.

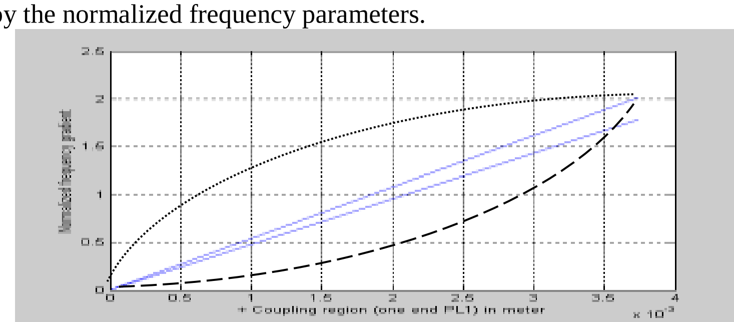

Figure 2. ∇V is 1.8 at first of PL_1 and 2 at end of PL_1 for zero coupling region

When normalized frequency is integrated, as shown by the straight lines in Figure 3, it describes the first PL_1 to be higher than that of end PL_1. The coupling region is set at zero position and lets the curves move to the initial V at 3.75 x 10^{-3}mm. This phenomena express that the change of each parameter of V which is set nearly linear although the actual changes are not as obvious. One of the parametric values of V is evaluated based on linear assumption that results in a significant dependence when changing to both gradient and integral of V is radius of core by the order of 10^{-3}. Refractive indices and wavelength do not have very necessary linear impact since the refractive indices and wavelength difference are kept by the order of 10^{-6} and 10^{-9} respectively. Therefore the linear effect is maintained and in retaining the mode at LP_{01}. Changing V from the initial value to the coupling length position, 2.45 to 0.4325-0.6646 will implicitly explain that the power transmission will be reduced along the coupling region. This result refers the power loss to the reflection and absorption as being inversely proportional to the factor of V^2 where $P \sim 1/V^2$. Assuming that V does not linearly or exponentially decrease or increase both the V gradient and integration. If it is affected by a function factor of $V=V(1-e^{\alpha})$ as shown in the dotted lines, where α is constant, then the radius geometry is not proportional to the speed of the pulling length. If it is by factor of $V=V(1-e^{-\alpha})$ as shown in the dashed lines in Figure 6 then it means that the fibers are not precisely heated at the center whereas the gradient of the refractive indices will be close to the factor of 10^{-3}. On the contrary, both reasons are not that acceptable owing to the fact that the mechanical process of fabrication is fixed and the radius change is much significant than other parameter of normalized frequency.

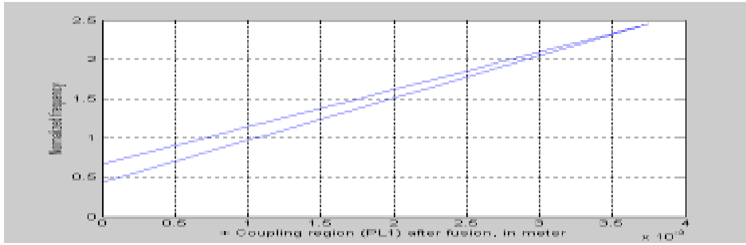

Figure 3. At zero coupling region, V integration gives 0.66 at first and 0.43 at end of PL_1

Table 1 describes details of parametric value changes along coupling region. A validation of code results is maintained by initial and final *V*, while at coupling region (excluding the coupling length) it is assumed to have constant linear changes.

Table 1. Normalized frequency of coupled SMF-28e®

Parameter	∇ at first of PL_1	∇ at end of PL_1	∇ at PL_2	∇ at first of PL_3	∇ at end of PL_3
(*z* position)	0 ⟶	$3.75\text{x}10^{-3}$ ⟶	$5.42\text{x}10^{-3}$ ⟶		$7.5\text{x}10^{-3}$mm ⟶
λ	0 to $4.5\text{x}10^{-9}$ (+)	0 to $7\text{x}10^{-9}$ (+)	0	0 to $-7\text{x}10^{-9}$ (-)	0 to $-4.5\text{x}10^{-9}$ (-)
a	0 to 1.5 (+)	0 to 2 (+)	0	0 to -2 (-)	0 to -1.5 (-)
n_1,n_2	0 to $-2.55\text{x}10^{-7}$ (-)	0 to $-3.48\text{x}10^{-7}$(-)	0	0 to $3.48\text{x}10^{-7}$(+)	0 to $2.55\text{x}10^{-7}$(+)
$V\|_{+,-}$	0 to 1.8 (+)	0 to 2 (+)	0	0 to -2 (-)	0 to -1.8 (-)
$\int V$	0.6646	0.4325	-	0.4325	0.6646

Initial SMF-28e® $V= V_1 = 2.4506$; n_1=1.4677 and n_2=1.4624; and a= 4.1e-6m
After Fusion $V= V_2 = 0.9761- 0.3353$; n_1=1.4623-1.4640; n_2=1.4556-1.4577; and a= 0.5 to 1.5e-6m
(*V*, V_1 and V_2 values are calculated from refractive indices known. The symbol of (+) and (-) indicates positive and negative gradient respectively and deal with along each z direction 0 to 3.75e-3mm).

VI. CONCLUSION

Coupling ratio range at 1 to 75% as corresponding to the range of coupling coefficient 0.6-0.9/mm has successfully developed the model of normalized frequency gradient and its integration along the coupling region. Increasing coupling ratio is done by increasing the coupling length and not the coupling coefficient, so that the normalized frequency gradient will gives parametric changes over the power transmission to the fiber. The core radius is much more affected to *V* compared to the refractive indices and the wavelength even though they change linearly.

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